Algebra II
Dr. Paul L. Bailey

Homework 1122
Monday, November 22, 2021

Name:

Due Tuesday, November 23, 2021. Do not copy. Do not write anything you do not understand.

Definition 1. Let f be a function defined on $\mathbb{R}$.
We say that f is *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$.
We say that f is *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$.

Proposition 1. *A polynomial is even if and only if all of its monomial summands have even degree. A polynomial is odd if and only if all of its monomial summands have odd degree.*

Problem 1. Determine if the given polynomial is even, odd, or neither.

(a) $x^2 - 25$

(b) $8x - 3x^2$

(c) $x^3 - 7x$

(d) $(x-1)(x-3)^2(x-5)^3$

(e) $x^5 - x^3 + 1$

(f) $2x$

Problem 2. Let $f(x) = x^2 + bx + c$, where $b, c \in \mathbb{R}$. Suppose $f(-7 + 2i) = 0$.
Find b and c.

Problem 3. Let $f(x) = x^3 + bx^2 + cx + d$, where $b, c, d \in \mathbb{R}$. Suppose $f(3) = 0$ and $f(5 + i) = 0$.
Find b, c, and d.

Definition 2. Let $A \subset \mathbb{R}$. We say that A is *connected* if, for every $a, b \in A$, if $a < x < b$, then $x \in A$.

Definition 3. An *interval* is a connected set of real numbers which contains more than one element. There are nine types:

- $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$
- $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$
- $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$
- $(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$
- $(-\infty, b] = \{x \in \mathbb{R} \mid x \leq b\}$
- $(-\infty, b) = \{x \in \mathbb{R} \mid x < b\}$
- $[a, \infty) = \{x \in \mathbb{R} \mid a \leq x\}$
- $(a, \infty) = \{x \in \mathbb{R} \mid a < x\}$
- $(-\infty, \infty) = \mathbb{R}$

Problem 4. Write the following sets in interval notation.

(a) The set of all real numbers between 3 and 11, including 3 and 11.

(b) The set of all real numbers between -2 and $3.\overline{3}$, including -2 but excluding $3.\overline{3}$.

(c) The set of all real number strictly less than 123.

(d) $\{x \in \mathbb{R} \mid x \geq \pi\}$

(e) $\{x \in \mathbb{R} \mid x^2 \leq 25\}$

Problem 5. Solve the following inequalities. Write the solution in interval notation.

(a) $x \geq 12$

(b) $x^2 \leq 7$

(c) $(x - 8)^2 \leq 0$

(d) $x^2 - 2x \geq 15$

(e) $x^2 - 2x \leq 15$